2. KOLOKVIJ – MATEMATIKA 2 – OGLEDNI PRIMJERAK – 2. DIO

1. Mora vrijediti: a) $8x-6y+x^2+y^2 \geq 0$ (zbog korijena)

   b) $xy>0$ (zbog logaritma)

a) $\underbrace{x^2+8x+16}+\underbrace{y^2-6y+9}-16-9 \geq 0$

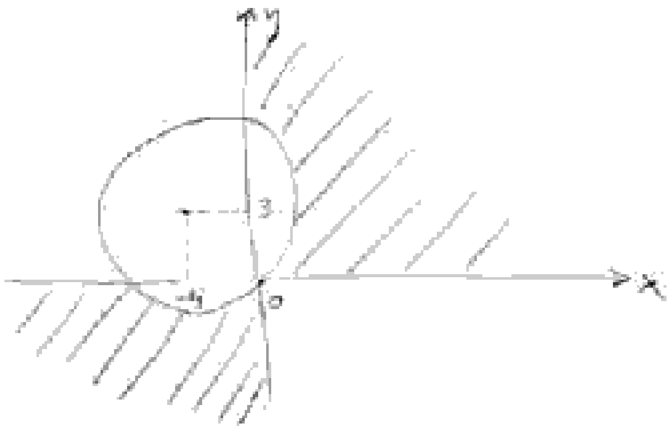
$(x+4)^2+(y-3)^2 \geq 5^2 \Rightarrow$ područje izvan kružnice sa središtem u $(-4,3)$ i radijusom $r=5$, uključujući i kružnicu

b) $xy>0 \Rightarrow x>0, y>0$ (1. kvadrant)

ili

$x<0, y<0$ (2. kvadrant)

Slika:



2. i) $\frac{\partial f}{\partial x}(x,y)=y\cdot\frac{1}{2\sqrt{x}}-1$

$\frac{\partial f}{\partial y}(x,y)=\sqrt{x}\cdot 1-2y+6$

ii) $\frac{\partial^2 f}{\partial x^2}(x,y)=\frac{1}{2}y\cdot\frac{-1}{2}\cdot x^{-\frac{3}{2}}=\frac{-1}{4}\cdot\frac{y}{\sqrt{x^3}}=\frac{-y}{4\sqrt{x^3}}$

$\frac{\partial^2 f}{\partial y\partial x}(x,y)=\frac{\partial^2 f}{\partial x\partial y}(x,y)=\frac{1}{2\sqrt{x}}$

$\frac{\partial^2 f}{\partial y^2}(x,y)=-2$

3. (i) $z = y\sqrt{x} - y^2 - x + 6y$

$(4,1,z_0) \Rightarrow z_0 = \sqrt{4} - 1^2 - 4 + 6\cdot 1 = 3$

$$\frac{\partial f}{\partial x}(x_0,y_0)\cdot(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)\cdot(y-y_0) - (z-z_0) = 0$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{y}{2\sqrt{x}} - 1 \Rightarrow \frac{\partial f}{\partial x}(4,1) = \frac{1}{2\sqrt{4}} - 1 = \frac{1}{4} - 1 = \frac{-3}{4}$$

$$\frac{\partial f}{\partial y}(x,y) = \sqrt{x} - 2y + 6 \Rightarrow \frac{\partial f}{\partial y}(4,1) = \sqrt{4} - 2\cdot 1 + 6 = 6$$

$$\Rightarrow \frac{-3}{4}(x-4) + 6(y-1) - (z-3) = 0 \quad /\cdot 4$$

$$-3x + 12 + 24y - 24 - 4z + 12 = 0 \quad /\cdot(-1)$$

$$3x - 24y + 4z = \cancel{12} - \cancel{24} + \cancel{12} \Rightarrow \boxed{3x - 24y + 4z = 0}$$

(ii) $$f(x_0+\Delta x, y_0+\Delta y) \approx \frac{\partial f}{\partial x}(x_0,y_0)\cdot\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\cdot\Delta y + f(x_0,y_0)$$

$x_0 = 4,\ \Delta x = -0.1$

$y_0 = 1,\ \Delta y = 0.1$

$$\Rightarrow f(3.9, 1.1) \approx \underbrace{f(4,1)}_{=3} + \underbrace{\frac{\partial f}{\partial x}(4,1)}_{=-\frac{3}{4}}\cdot(-0.1) + \underbrace{\frac{\partial f}{\partial y}(4,1)}_{=6}\cdot(0.1) =$$

$$= 3 - \frac{3}{4}\cdot\frac{-1}{10} + \overset{3}{\cancel{6}}\cdot\frac{1}{\underset{5}{\cancel{10}}} = 3 + \frac{3}{40} + \frac{3}{5} = 3 + \frac{3+24}{40}$$

$$\Rightarrow f(3.9, 1.1) \approx 3 + \frac{27}{40}$$

4. (i) $$\frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial y}(x,y) = 0$$

$$\Rightarrow \begin{cases} \dfrac{y}{2\sqrt{x}} - 1 = 0 \Rightarrow y = 2\sqrt{x} \\ \sqrt{x} - 2y + 6 = 0 \Rightarrow \sqrt{x} - 2\cdot 2\sqrt{x} + 6 = 0 \end{cases}$$

$$-3\sqrt{x} = -6 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$$

$$y = 2\sqrt{4} = 4$$

$$H = \begin{bmatrix} \dfrac{-y}{4\sqrt{x^3}} & \dfrac{1}{2\sqrt{x}} \\ \dfrac{1}{2\sqrt{x}} & -2 \end{bmatrix} \text{ - vidi 2. zad. (ii)}$$

Uvrstimo u H (4,4):

$$H_{(4,4)} = \begin{bmatrix} \frac{-4}{4\sqrt{4^3}} & \frac{1}{2\sqrt{4}} \\ \frac{1}{2\sqrt{4}} & -2 \end{bmatrix} = \dots = \begin{bmatrix} -\frac{1}{8} & \frac{1}{4} \\ \frac{1}{4} & -2 \end{bmatrix}$$

$$\Delta = -\frac{1}{8}\cdot(-2) - \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{4} - \frac{1}{16} = \frac{7}{16} > 0$$

$A = -\frac{1}{8} < 0 \Rightarrow$ u (4,4) se postiže lok. maksimum i iznosi:

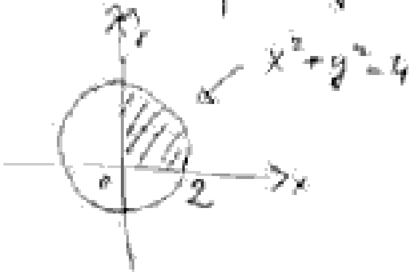
$$f(4,4) = 4\sqrt{4} - 4^2 - 4 + 6\cdot 4 = \dots = \underline{12}$$

(ii) Treba zapravo pokazati da $g(x,y)$ nema niti kandidata za lokalne ekstreme.

$$\boxed{\begin{aligned} \frac{\partial g}{\partial x}(x,y) &= \frac{1}{xy}\cdot y = \frac{1}{x} = 0 \\ \frac{\partial g}{\partial y}(x,y) &= \frac{1}{xy}\cdot x = \frac{1}{y} = 0 \end{aligned}}$$

Ovaj sustav očito nema rješenja. Kako nema kritičnih točaka, tako nema niti lokalnih ekstrema.

5. (i) Gledamo područje integracije:



po x: $x=0, x=2$

po y: $y=0$, a odozgo omeđeno kružnicom $x^2+y^2=4$

$\Rightarrow y^2 = 4-x^2$

$\Rightarrow y = \sqrt{4-x^2}$, pa

imamo $$\int_0^2 \int_0^{\sqrt{4-x^2}} xy^2\,dy\,dx.$$ No, mogli smo područje integracije promatrati i na način da y bude nezavisna varijabla, pa bi imali: granice po y: $y=0, y=2$

granice po x: $x=0$, $x^2+y^2=4$

$\Rightarrow x = \sqrt{4-y^2}$, pa

$$\int_0^2 \int_0^{\sqrt{4-y^2}} xy^2\,dx\,dy$$

(ii) Lakše će nam biti riješiti drugi integral:

$$\int_0^2 \int_0^{\sqrt{4-y^2}} xy^2\,dx\,dy = \int_0^2 \left(\frac{x^2y^2}{2}\right)\Big|_0^{\sqrt{4-y^2}} dy =$$

$$= \int_0^2 \left(\frac{(\sqrt{4-y^2})^2 y^2}{2} - \underbrace{\frac{0^2 \cdot y^2}{2}}_{=0}\right) dy =$$

$$= \int_0^2 \frac{(4-y^2)y^2}{2}\,dy = \frac{1}{2}\int_0^2 (4y^2 - y^4)\,dy =$$

$$= \frac{1}{2}\left(\left(4\cdot\frac{y^3}{3}\right)\Big|_0^2 - \left(\frac{y^5}{5}\right)\Big|_0^2\right) =$$

$$= \frac{1}{2}\left(4\cdot\frac{2^3}{3} - \underbrace{4\cdot\frac{0^3}{3}}_{=0} - \left(\frac{2^5}{5} - \underbrace{\frac{0^5}{5}}_{=0}\right)\right) =$$

$$= \frac{1}{2}\cdot\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{\cancel{32}^{16}}{\cancel{2}}\left(\frac{1}{3} - \frac{1}{5}\right) = 16\cdot\frac{5-3}{15} =$$

$$= 16\cdot\frac{2}{15} = \underline{\frac{32}{15}}\,\Big|.$$